THREE QUBIT SYSTEM FOR KICKED TOP MODEL

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Master of Science In Physics

by

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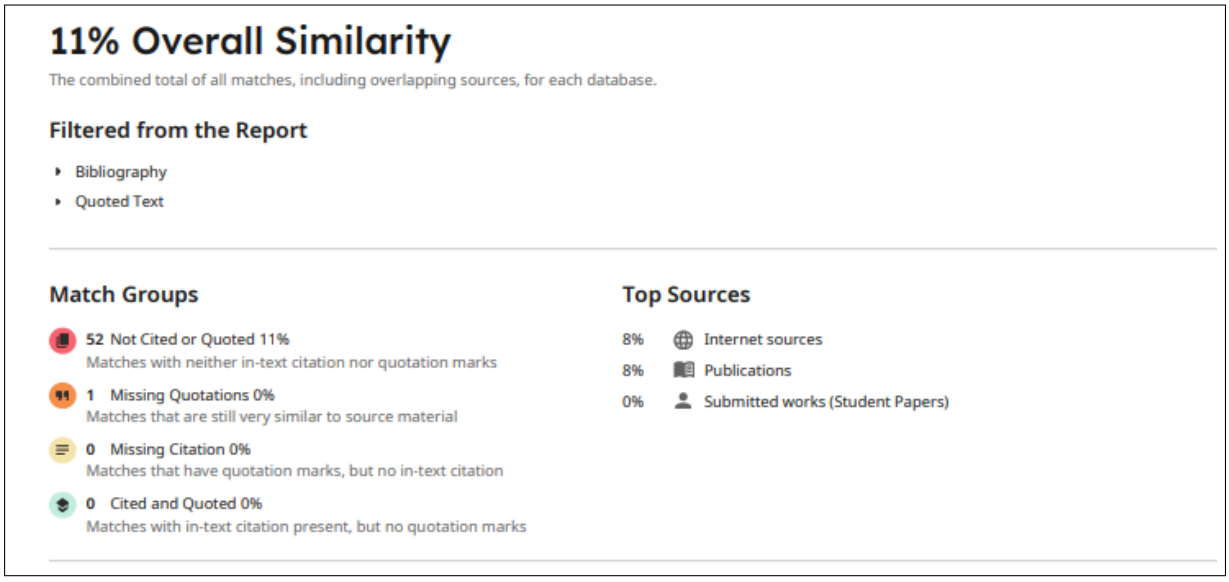

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ABSTRACT

This thesis investigates the behavior of a simple quantum system [**?**] known as the kicked top model when subjected to repeated disturbances over time. Specifically, we study a spatial variant of this model using a three-qubit setup, where the qubits interact either symmetrically or asymmetrically. To analyze the system's evolution, we apply Floquet theory [2]- a powerful framework for understanding periodically driven systems. Key features such as symmetry, quantum chaos, and entanglement(which quantifies the interconnection between qubits) are explored in detail. By varying the interaction patterns among the qubits, we observe transitions from regular, predictable dynamics to chaotic, complex behavior. The study also compares the quantum dynamics with classical expectations to highlight quantum-classical correspondence. To simplify the analysis and uncover essential properties, we employ special mathematical techniques and make use of well-known quantum states like the GHZ and W states. Overall, this work deepens our understanding of complex quantum phenomena in small systems, with potential implications for quantum computing and quantum simulation technologies.

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Chapter 1

Introduction

Floquet theory [4] helps analyze a quantum system influenced by the forces [**?**] in a repeating pattern. As the system's energy evolves periodically, the traditional study method is ineffective. In Floquet theory, the Floquet operator tool comes in handy to explore time-periodic Hamiltonians. According to the Schrödinger equation, given below, it tells how the system changes over time:

$$i\hbar\frac{d}{dt}|\psi(t)\rangle = H(t)|\psi(t)\rangle. \tag{1.1}$$

It helps to find how the state $|\psi(t)\rangle$ changes with time when the Hamiltonian repeats.

1.1 What is Floquet's Theorem? [4, 13]

According to Floquet's theorem, a system with repeating energies can be expressed as:

$$|\psi(t)\rangle = e^{-i\varepsilon t/\hbar}|\phi(t)\rangle. \tag{1.2}$$

Here, ε is *quasienergy*.
$|\phi(t)\rangle$: Function repeating after every t time.

1.2 The Floquet Operator [2]

The Floquet operator considers the system at once every period T, which shows how the state changes in one full period, which can be written as :

$$U(T) = \mathcal{T}\exp\left(-\frac{i}{\hbar}\int_0^T H(t')dt'\right). \tag{1.3}$$

This operator is unitary, so the eigenstates:

$$U(T)|u_\alpha\rangle = e^{-i\varepsilon_\alpha T/\hbar}|u_\alpha\rangle. \tag{1.4}$$

Here, $|u_\alpha\rangle$ Floquet states and ε_α : Quasienergies.

1.3 Time Evolution Using Floquet Theory

To find the system's state after n periods, [**?**] we should know the Floquet operator,

$$|\psi(nT)\rangle = [U(T)]^n |\psi(0)\rangle. \tag{1.5}$$

For regular time intervals,

$$|\psi(t)\rangle = \sum_\alpha c_\alpha e^{-i\varepsilon_\alpha t/\hbar} |\psi_\alpha(t)\rangle, \tag{1.6}$$

where $c_\alpha = \langle \phi_\alpha(0)|\psi(0)\rangle$

1.4 Why Floquet Theory is Important

The uses of Floquet theory are as follows,

- Quantum simulators in labs.
- Study of time crystals.
- Study of quantum chaos [5, 6, 10].

Chapter 2

The Kicked Top Model

2.1 What is the Kicked Top? [1]

The kicked Top [**?**] is a basic model of quantum physics used to explore how complexity can arise in a small system due to repeated disturbances. Imagine a spinning toy top that you tap at regular intervals- each tap slightly alters its motion. In quantum mechanics, this concept helps us study how continuous kicks can lead to unexpected and even chaotic behavior in a system.

2.2 How is the Energy expressed? (Hamiltonian)

To represent the energy of the system, we use a mathematical tool called the Hamiltonian, which explains how the system evolves over time. In the kicked top model, the Hamiltonian is divided into two key components:

- One represents continuous, smooth rotation
- The other part captures the effects of sudden, repeated kicks.

The formula is given as:

$$H(t) = \frac{\kappa}{2j} j_z^2 \sum_n \delta(t - nT) + \frac{\pi}{2} j_y. \tag{2.1}$$

here,

- j_z^2 explains how the top twists during the kicks.
- j_y explains the smooth spinning.
- κ kick strength
- j size of the spin
- $\delta(t - nT)$ explains that the kicks happen at a regular time interval

2.3 How the System Changes Over Time? (Floquet Operator) [4, 13]

Rather than observing the system continuously, we can examine it immediately after each kick. The tool used for this is known as the Floquet operator. It captures how the system evolves from one kick to the next.

$$U = \exp\left(-i\frac{\kappa}{2j}J_z^2\right) \cdot \exp\left(-i\frac{\pi}{2}J_y\right). \tag{2.2}$$

We can analyze how the system behaves over time by applying it repeatedly.

$$|\psi_n\rangle = U^n|\psi_0\rangle.$$

here, $|\psi_0\rangle$ is the initial state, and $|\psi_n\rangle$ is the state after n kicks.

2.4 Equal Interactions (Symmetric or All-to-All) [3, 12]

In the kicked top model, each part of the system interacts [**?**]equally with every other part- a feature known as all-to-all interaction. This uniform interaction adds a sense of balance to the system, but also increases its complexity. As a result, the system can exhibit a wide range of behaviors, from orderly and predictable motion to fully chaotic dynamics.

Chapter 3

Modified Interaction Structure

3.1 What Changes When Coupling is Not Uniform?

In the standard kicked top model, all parts of the system interact equally with one another, which is known as **all-to-all** interaction. But what happens if we alter this setup? When the interactions become **non-uniform**, some parts of the system interact more strongly than others; this shift breaks the system's symmetry, making it less uniform and more structured, similar to a chain or a network with specific links. While this can lead to richer and more intriguing behavior, it also makes the system more difficult to analyze and predict.

3.2 How Symmetry is Affected?

When all parts of a system interact equally, it possesses a symmetry known as **permutation symmetry**- meaning the system remains unchanged if any two parts are swapped. However, when the coupling between parts becomes non-uniform, this symmetry is disrupted. The system no longer remains invariant under all exchanges. While it might still retain some hidden or partial symmetries, this loss of full symmetry often makes the system more complex, potentially shifting it from being exactly solvable to unsolvable, or at least much harder to solve analytically.

3.3 Solvability and Chaos [6, 15]

In many situations, breaking the system's symmetry leads to more chaotic behavior. Finding exact solutions becomes more difficult, often requiring numerical simulations instead. However, there are cases where a clever approach can help the system with altered interactions that might be transformed or mapped onto another well-understood system.

3.4 Reduction to Spin Chains [2]

If interaction strength varies with position, such as in nearest-neighbor interactions, the system can resemble a quantum spin chain. These are well-studied models characterized by local interactions, where each component interacts only with its immediate neighbor. This mapping is not always possible, but when it is, it helps us understand the system better.

3.5 Why This is Important

Modifying the interaction pattern allows scientists to investigate novel behaviors such as:

- The influence of symmetry on entanglement and chaos.
- The emergence of new types of order.
- The transition of systems from integrable and solvable to non-integrable and chaotic regimes.

These play an important role in **quantum simulation**,**many-body physics**, and **quantum computing**.

Chapter 4

Parity Symmetry and Subspace Reduction

4.1 What is Parity Symmetry?

In quantum systems, symmetry implies that the system's behavior remains unchanged under certain transformations. Parity symmetry is one example of such a symmetry. In the case of a 3-qubit system, a specific parity operator can be defined to characterize this symmetry [1].

$$P = \sigma_1^y \otimes \sigma_2^y \otimes \sigma_3^y. \tag{4.1}$$

Applying the Pauli-Y matrix to each qubit. If the unitary operator of the system U commutes with this operator, then

$$[U, P] = 0. \tag{4.2}$$

This implies that the system is symmetric under this operation.

4.2 Why Commutation is Important?

When the operator U commutes with the symmetry operator, it means we can use the symmetry to simplify the problem. The full Hilbert space can be broken down into smaller, independent subspaces. This makes it easier to analyze and understand the system's behavior [9].

4.3 GHZ and W states and Basis States

Let's use the special entangled states expressed as a new basis that follows this symmetry.

GHZ State:

$$|GHZ\rangle = \frac{1}{\sqrt{2}}(|000\rangle + |111\rangle),$$

W States:

$$|W\rangle = \frac{1}{\sqrt{3}}(|001\rangle + |010\rangle + |100\rangle),$$
$$\overline{W} = \frac{1}{\sqrt{3}}(|110\rangle + |101\rangle + |011\rangle,$$

4.4 Subspace Reduction [2]

By applying the parity operator P full 8-dimensional Hilbert space of a 3-qubit system can be decomposed into smaller subspaces. For example, certain states may belong to subspaces with parity $+1$, while others may fall into two distinct subspaces with parity -1. This decomposition allows us to analyze each subspace individually, making the system easier to study.

4.5 Symmetric or Parity Basis

The kicked top often shows a kind of symmetry—like even vs. odd parity—which helps us **simplify** the matrix form of U. If we choose a symmetry-adapted basis, we can write U as a smaller matrix acting in this reduced space.

Chapter 5

Entanglement Measures [11, 14]

5.1 What is Entanglement? [8, 9]

Entanglement is a unique feature of quantum mechanics. It means different parts of quantum systems are so connected that moving one part tells you something about the other. We use different methods to measure how strong these connections are.

5.2 Linear Entropy (1-Qubit Entanglement)

To measure how tangled a single qubit is with the rest of the system, we use linear entropy. This involves first calculating the reduced density matrix ρ_1 by tracing out the other qubits.

$$S_L = 1 - \text{Tr}(\rho_1^2). \tag{5.1}$$

If $S_L = 0$, the qubit is pure. If $S_L > 0$, the qubit is entangled.

5.3 Concurrence (2-Qubit Entanglement) [1]

Concurrence tells about the entanglement of the two qubits. Starting with the reduced density matrix ρ_{12}, we get

$$\tilde{\rho} = (\sigma^y \otimes \sigma^y)\rho_{12}^*(\sigma^y \otimes \sigma^y) \tag{5.2}$$

$$R = \rho_{12}\tilde{\rho} \tag{5.3}$$

Finding the square roots of the eigenvalues of R, such that $\lambda_1, \lambda_2, \lambda_3, \lambda_4$ (in decreasing order), then we get,

$$C = \max(0, \lambda_1 - \lambda_2 - \lambda_3 - \lambda_4). \tag{5.4}$$

Chapter 6

Objectives

- To find the values of the kick strengths κ_0 and κ such that the system will show the periodic behavior.
- To study κ_0 as the kick strength between qubit 1 and qubit 2, and between qubit 2 and qubit 3.
- To study κ as the kick strength between qubit 3 and qubit
- To understand the effect of the unequal kick strength on the kicked top model of the three-qubit system via Linear Entropy.

Chapter 7

Methodology

9.1 Formulation of the Hamiltonian & Unitary Operator

The three-qubit kicked top model has the Hamiltonian, with the kick strengths κ_0 for the interactions between qubit 1 and 2, and qubit 2 and 3, and κ for qubit 3 and 1, is written as:

$$H = \frac{1}{3}\left(\kappa_0\sigma_1^z\sigma_2^z + \kappa_0\sigma_2^z\sigma_3^z + \kappa\sigma_3^z\sigma_1^z\right)\sum_{n=\infty}^{\infty}\delta(t-n\tau) + \frac{\pi}{4\tau}\sum_{l=1}^{3}\sigma_1^y, \qquad (7.1)$$

where $\sigma_l^{x,y,z}$ are Pauli matrices for the l-th qubit, τ = **period between kicks**, and the rotation term applies a $\pi/2$ rotation about the y-axis.

The corresponding Floquet Operator to show the evolution of the system between kicks,

$$\mathcal{U} = \exp\left[-i\frac{1}{6}\left(\kappa_0\sigma_1^z\sigma_2^z + \kappa_0\sigma_2^z\sigma_3^z + \kappa\sigma_3^z\sigma_1^z\right)\right]\exp\left[-i\frac{\pi}{4}\left(\sigma_1^y + \sigma_2^y + \sigma_3^y\right)\right]. \qquad (7.2)$$

This operator operates within the permutation-symmetric subspace, aiding the study of quantum signatures of chaos and thermalisation.

We verify that the Floquet operator $U = U_{\text{int}}U_{\text{rot}}$ commutes with the parity operator $P = \sigma_y^1 \otimes \sigma_y^2 \otimes \sigma_y^3$, where:

$$U_{\text{int}} = \exp\left[-i\frac{1}{6}\left(\kappa_0\sigma_z^1\sigma_z^2 + \kappa_0\sigma_z^2\sigma_z^3 + \kappa\sigma_z^3\sigma_z^1\right)\right], \quad U_{\text{rot}} = \exp\left[-i\frac{\pi}{4}\left(\sigma_y^1 + \sigma_y^2 + \sigma_y^3\right)\right]$$

Step 1: Compute $[U_{\text{rot}}, P]$

$$U_{\text{rot}} = \exp\left(-i\frac{\pi}{4}\sigma_y^1\right) \otimes \exp\left(-i\frac{\pi}{4}\sigma_y^2\right) \otimes \exp\left(-i\frac{\pi}{4}\sigma_y^3\right),$$
$$P = \sigma_y^1 \otimes \sigma_y^2 \otimes \sigma_y^3.$$

For each Qubit σ_y commutes with $\exp\left(-i\frac{\pi}{4}\sigma_y\right)$ since $[\sigma_y, \sigma_y] = 0$. Thus:

$$U_{\text{rot}}P = \left(\sigma_y^1 \exp\left(-i\frac{\pi}{4}\sigma_y^1\right)\right) \otimes \left(\sigma_y^2 \exp\left(-i\frac{\pi}{4}\sigma_y^2\right)\right) \otimes \left(\sigma_y^3 \exp\left(-i\frac{\pi}{4}\sigma_y^3\right)\right).$$

$$PU_{\text{rot}} = \left(\sigma_y^1 \exp\left(-i\frac{\pi}{4}\sigma_y^1\right)\right) \otimes \left(\sigma_y^2 \exp\left(-i\frac{\pi}{4}\sigma_y^2\right)\right) \otimes \left(\sigma_y^3 \exp\left(-i\frac{\pi}{4}\sigma_y^3\right)\right).$$

$$[U_{\text{rot}}, P] = U_{\text{rot}}P - PU_{\text{rot}} = 0.$$

Step : Compute $[U_{\text{int}}, P]$

$$U_{\text{int}} = \exp\left[-i\frac{1}{6}\left(\kappa_0\sigma_z \otimes \sigma_z \otimes I + \kappa_0 I \otimes \sigma_z \otimes \sigma_z + \kappa\sigma_z \otimes I \otimes \sigma_z\right)\right].$$

Let $H_{\text{int}} = \frac{1}{6}\left(\kappa_0\sigma_z \otimes \sigma_z \otimes I + \kappa_0 I \otimes \sigma_z \otimes \sigma_z + \kappa\sigma_z \otimes I \otimes \sigma_z\right)$.

Check $[H_{\text{int}}, P] = 0$,

$$P(\sigma_z\otimes\sigma_z\otimes I)P = (\sigma_y\sigma_z\sigma_y)\otimes(\sigma_y\sigma_z\sigma_y)\otimes(\sigma_y I\sigma_y) = (-\sigma_z)\otimes(-\sigma_z)\otimes I = \sigma_z\otimes\sigma_z\otimes I.$$

$$P(I\otimes\sigma_z\otimes\sigma_z)P = (\sigma_y I\sigma_y)\otimes(\sigma_y\sigma_z\sigma_y)\otimes(\sigma_y\sigma_z\sigma_y) = I\otimes(-\sigma_z)\otimes(-\sigma_z) = I\otimes\sigma_z\otimes\sigma_z.$$

$$P(\sigma_z\otimes I\otimes\sigma_z)P = (\sigma_y\sigma_z\sigma_y)\otimes(\sigma_y I\sigma_y)\otimes(\sigma_y\sigma_z\sigma_y) = (-\sigma_z)\otimes I\otimes(-\sigma_z) = \sigma_z\otimes I\otimes\sigma_z.$$

Since $PH_{\text{int}}P = H_{\text{int}}$, $[H_{\text{int}}, P] = 0$, so $[U_{\text{int}}, P] = 0$.
Hence, $[U, P]$=0

Formation of the Eigenbasis states

Given the Hamiltonian,

$$H = \frac{1}{3}\left(\kappa_0\sigma_1^z\sigma_2^z + \kappa_0\sigma_2^z\sigma_3^z + \kappa\sigma_3^z\sigma_1^z\right). \tag{7.3}$$

σ_1^z and σ_2^z are in Permutation Symmetry (invariant)

$$|\phi_1\rangle = |00\rangle, \tag{7.4}$$
$$|\phi_2\rangle = |11\rangle, \tag{7.5}$$
$$|\phi_3\rangle = \frac{1}{\sqrt{2}}\left(|10\rangle + |01\rangle\right), \tag{7.6}$$

Appending $|0\rangle$ from the right side, we get:

$$|\phi_1\rangle = |000\rangle, \tag{7.7}$$
$$|\phi_2\rangle = |110\rangle, \tag{7.8}$$
$$|\phi_3\rangle = \frac{1}{\sqrt{2}}\left(|100\rangle + |010\rangle\right), \tag{7.9}$$

Appending $|1\rangle$ from the right side, we get,

$$|\phi_1*\rangle = |001\rangle, \tag{7.10}$$
$$|\phi_2*\rangle = |111\rangle, \tag{7.11}$$
$$|\phi_3*\rangle + \frac{1}{\sqrt{2}}\left(|101\rangle + |011\rangle\right), \tag{7.12}$$

Now Applying

$$\left(\otimes_{l=1}^{3}\sigma_l^y\right) \tag{7.13}$$

on all the eigenstates such that we get the Parity Eigenstates, such that: $P = \otimes_{l=}^{3}\sigma_l^y = \sigma_1^y \otimes \sigma_2^y \otimes \sigma_3^y$ on the states we get

$$P|\phi_1\rangle = P|000\rangle = -i|111\rangle, \tag{7.14}$$
$$P|\phi_2\rangle = P|110\rangle = -i|001\rangle, \tag{7.15}$$
$$P|\phi_3\rangle = P\frac{1}{\sqrt{2}}\left(|100\rangle + |010\rangle\right) = i|\phi_3*\rangle, \tag{7.16}$$
$$P|\phi_1*\rangle = P|001\rangle = -i|110\rangle, \tag{7.17}$$
$$P|\phi_2*\rangle = P|111\rangle = -i|000\rangle, \tag{7.18}$$
$$P|\phi_3*\rangle = P\frac{1}{\sqrt{2}}\left(|101\rangle + |011\rangle\right) = -i|\phi_3\rangle, \tag{7.19}$$

Positive Parity Subspace $P|\psi\rangle = +|\psi\rangle$,

$$|\psi_1^+\rangle = \frac{1}{\sqrt{2}}\left(|000\rangle - i|111\rangle\right), \tag{7.20}$$
$$|\psi_2^+\rangle = \frac{1}{\sqrt{2}}\left(|110\rangle - i|001\rangle\right), \tag{7.21}$$
$$|\psi_3^+\rangle = \frac{1}{2}\left(|100\rangle + |010\rangle - i|101\rangle - i|011\rangle\right), \tag{7.22}$$

Negative Parity Subspace $P|\psi\rangle = -|\psi\rangle$,

$$|\psi_1^-\rangle = \frac{1}{\sqrt{2}}\left(|000\rangle + i|111\rangle\right), \tag{7.23}$$
$$|\psi_2^-\rangle = \frac{1}{\sqrt{2}}\left(|110\rangle + i|001\rangle\right), \tag{7.24}$$
$$|\psi_3^-\rangle = \frac{1}{2}\left(|100\rangle + |010\rangle + i|101\rangle + i|011\rangle\right). \tag{7.25}$$

7.1 Block-Diagonalized Floquet Matrices

The Floquet operator for the modified three-qubit kicked top model is:

$$\mathcal{U} = \exp\left[-i\frac{1}{6}\left(\kappa_0\sigma_1^z\sigma_2^z + \kappa_0\sigma_2^z\sigma_3^z + \kappa\sigma_3^z\sigma_1^z\right)\right]\exp\left[-i\frac{\pi}{4}\left(\sigma_1^y + \sigma_2^y + \sigma_3^y\right)\right].$$

Since $[\mathcal{U}, P] = 0$ with $P = \otimes_{l=1}^{3}\sigma_1^y$, $\mathcal{U}$ is block-diagonal in the positive and negative parity subspaces:

$$\mathcal{U} = \begin{pmatrix} \mathcal{U}_+ & 0 \\ 0 & \mathcal{U}_- \end{pmatrix}$$

Such that,

$$\mathcal{U}_+ = \begin{pmatrix} \langle\psi_1^+|\mathcal{U}|\psi_1^+\rangle & \langle\psi_1^+|\mathcal{U}|\psi_2^+\rangle & \langle\psi_1^+|\mathcal{U}|\psi_3^+\rangle \\ \langle\psi_2^+|\mathcal{U}|\psi_1^+\rangle & \langle\psi_2^+|\mathcal{U}|\psi_2^+\rangle & \langle\psi_2^+|\mathcal{U}|\psi_3^+\rangle \\ \langle\psi_3^+|\mathcal{U}|\psi_1^+\rangle & \langle\psi_3^+|\mathcal{U}|\psi_2^+\rangle & \langle\psi_3^+|\mathcal{U}|\psi_3^+\rangle \end{pmatrix}.$$

$$\mathcal{U}_- = \begin{pmatrix} \langle\psi_1^-|\mathcal{U}|\psi_1^-\rangle & \langle\psi_1^-|\mathcal{U}|\psi_2^-\rangle & \langle\psi_1^-|\mathcal{U}|\psi_3^-\rangle \\ \langle\psi_2^-|\mathcal{U}|\psi_1^-\rangle & \langle\psi_2^-|\mathcal{U}|\psi_2^-\rangle & \langle\psi_2^-|\mathcal{U}|\psi_3^-\rangle \\ \langle\psi_3^-|\mathcal{U}|\psi_1^-\rangle & \langle\psi_3^-|\mathcal{U}|\psi_2^-\rangle & \langle\psi_3^-|\mathcal{U}|\psi_3^-\rangle \end{pmatrix}.$$

To compute the matrix elements we have to separately apply $\mathcal{U}_{\text{int}}$ and $\mathcal{U}_{\text{rot}}$ on the parity subspaces, To compute the matrix elements $\langle\psi_i^+|\mathcal{U}|\psi_j^+\rangle$

- Apply U_{rot} on three qubits we get:

$$U_{\text{rot}}|ijk\rangle = \frac{1}{\sqrt{8}} \sum_{i',j',k'=0,1} (-1)^{i'(1-i)+j'(1-j)+k'(1-k)} |i'j'k'\rangle. \tag{7.26}$$

  Then its action on each computational basis state is:

$$\begin{aligned}
U_{\text{rot}}|000\rangle &= \frac{1}{\sqrt{8}}(|000\rangle + |001\rangle + |010\rangle + |011\rangle + |100\rangle + |101\rangle + |110\rangle + |111\rangle),\\
U_{\text{rot}}|001\rangle &= \frac{1}{\sqrt{8}}(-|000\rangle + |001\rangle - |010\rangle + |011\rangle - |100\rangle + |101\rangle - |110\rangle + |111\rangle),\\
U_{\text{rot}}|010\rangle &= \frac{1}{\sqrt{8}}(-|000\rangle - |001\rangle + |010\rangle + |011\rangle - |100\rangle - |101\rangle + |110\rangle + |111\rangle),\\
U_{\text{rot}}|011\rangle &= \frac{1}{\sqrt{8}}(|000\rangle - |001\rangle - |010\rangle + |011\rangle + |100\rangle - |101\rangle - |110\rangle + |111\rangle),\\
U_{\text{rot}}|100\rangle &= \frac{1}{\sqrt{8}}(-|000\rangle - |001\rangle - |010\rangle - |011\rangle + |100\rangle + |101\rangle + |110\rangle + |111\rangle),\\
U_{\text{rot}}|101\rangle &= \frac{1}{\sqrt{8}}(|000\rangle - |001\rangle + |010\rangle - |011\rangle + |100\rangle - |101\rangle + |110\rangle - |111\rangle),\\
U_{\text{rot}}|110\rangle &= \frac{1}{\sqrt{8}}(|000\rangle + |001\rangle - |010\rangle - |011\rangle + |100\rangle + |101\rangle - |110\rangle - |111\rangle),\\
U_{\text{rot}}|111\rangle &= \frac{1}{\sqrt{8}}(-|000\rangle + |001\rangle + |010\rangle - |011\rangle - |100\rangle + |101\rangle + |110\rangle - |111\rangle),
\end{aligned}$$

- Apply U_{int} on three qubit we get:

$$\begin{aligned}
U_{\text{int}}|000\rangle &= e^{-i\frac{2\kappa_0+\kappa}{6}}|000\rangle, && (7.27)\\
U_{\text{int}}|111\rangle &= e^{-i\frac{2\kappa_0+\kappa}{6}}|111\rangle, && (7.28)\\
U_{\text{int}}|110\rangle &= e^{i\frac{\kappa}{6}}|110\rangle, && (7.29)\\
U_{\text{int}}|001\rangle &= e^{i\frac{\kappa}{6}}|001\rangle, && (7.30)\\
U_{\text{int}}|001\rangle &= e^{i\frac{\kappa}{6}}|100\rangle, && (7.31)\\
U_{\text{int}}|100\rangle &= e^{-i\frac{2\kappa_0-\kappa}{6}}|100\rangle, && (7.32)\\
U_{\text{int}}|101\rangle &= e^{-i\frac{2\kappa_0-\kappa}{6}}|101\rangle, && (7.33)\\
U_{\text{int}}|011\rangle &= e^{i\frac{\kappa}{6}}|011\rangle, && (7.34)\\
& && (7.35)
\end{aligned}$$

Computing the matrix element $\langle\psi_1^+|\mathcal{U}|\psi_1^+\rangle$ we get:

$$\langle\psi_1^+|\mathcal{U}|\psi_1^+\rangle = \frac{1+i}{4} e^{-i\frac{2\kappa_0+\kappa}{6}}.$$

Similarly, all the elements of the matrix are as follows: The full-size 3*3 matrix

$$\begin{pmatrix} (1+i)e^{-i\frac{2\kappa_0+\kappa}{6}} & (1+i)e^{i\frac{\kappa}{6}} & \frac{1}{\sqrt{2}}\left[(1-i)(e^{i\frac{\kappa}{6}}+e^{i\frac{2\kappa_0-\kappa}{6}})+(1+i)(e^{-i\frac{\kappa}{6}}+e^{i\frac{\kappa}{6}})\right] \\ (1+i)e^{i\frac{\kappa}{6}} & (1+i)e^{i\frac{\kappa}{6}} & \frac{1}{\sqrt{2}}\left[(1+i)(e^{i\frac{2\kappa_0-\kappa}{6}}+e^{i\frac{\kappa}{6}})+(1-i)(e^{-i\frac{\kappa}{6}}+e^{i\frac{\kappa}{6}})\right] \\ \frac{1}{\sqrt{2}}\left[(1-i)(e^{i\frac{\kappa}{6}}+e^{i\frac{2\kappa_0-\kappa}{6}})+(1+i)(e^{-i\frac{\kappa}{6}}+e^{i\frac{\kappa}{6}})\right] & \frac{1}{\sqrt{2}}\left[(1+i)(e^{i\frac{2\kappa_0-\kappa}{6}}+e^{i\frac{\kappa}{6}})+(1-i)(e^{-i\frac{\kappa}{6}}+e^{i\frac{\kappa}{6}})\right] & \frac{1}{2}\left[(1+i)(e^{i\frac{\kappa}{6}}+e^{-i\frac{\kappa}{6}})+(1-i)(e^{i\frac{2\kappa_0-\kappa}{6}}+e^{i\frac{\kappa}{6}})\right]. \end{pmatrix}.$$

7.2 Computation of the Negative Parity Floquet Matrix $\mathcal{U}_-$

7.2.1 Step 1: Compute U_{rot}

For a single qubit:

$$U_y = \exp\left(-i\frac{\pi}{4}\sigma^y\right) = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix}.$$

$$U_y|0\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle), \quad U_y|1\rangle = \frac{1}{\sqrt{2}}(|1\rangle - |0\rangle).$$

Thus, $U_{\text{rot}} = U_y \otimes U_y \otimes U_y$. For basis states:

$$U_{\text{rot}}|ijk\rangle = \frac{1}{\sqrt{8}} \sum_{i',j',k'=0,1} (-1)^{i'(1-i)+j'(1-j)+k'(1-k)}|i'j'k'\rangle.$$

[**?**] For Example:

$$U_{\text{rot}}|000\rangle = \frac{1}{\sqrt{8}} \sum_{i',j',k'=0,1} |i'j'k'\rangle, \quad U_{\text{rot}}|111\rangle = \frac{1}{\sqrt{8}} \sum_{i',j',k'=0,1} (-1)^{i'+j'+k'}|i'j'k'\rangle.$$

7.2.2 Step 2: Compute U_{int}

The Interaction Hamiltonian is :

$$H_{\text{int}} = \frac{1}{6}\left(\kappa_0\sigma_1^z\sigma_2^z + \kappa_0\sigma_2^z\sigma_3^z + \kappa\sigma_3^z\sigma_1^z\right).$$

The action on a computational basis states:

Element $\langle\psi_1^-|\mathcal{U}|\psi_1^-\rangle$

$$|\psi_1^-\rangle = \frac{1}{\sqrt{2}}(|000\rangle + i|111\rangle), \quad \langle\psi_1^-| = \frac{1}{\sqrt{2}}(\langle 000| - i\langle 111|).$$

$$U_{\text{rot}}|\psi_1^-\rangle = \frac{1}{\sqrt{16}} \sum_{i,j,k=0,1} \left[1 + i(-i)^{i+j+k}\middle|\, ijk\right\rangle]$$

$$\mathcal{U}|\psi_1^-\rangle = \frac{1}{\sqrt{16}}\left[(1-i)e^{-i\alpha}|000\rangle + (1+i)e^{i\beta}|001\rangle + (1+i)e^{i\gamma}|010\rangle + (1-i)e^{i\beta}|011\rangle\right.$$
$$\left.+(1+i)e^{i\beta}|100\rangle + (1-i)e^{i\delta}|101\rangle + (1-i)e^{i\beta}|110\rangle + (1+i)e^{-i\alpha}|111\rangle\right.$$

Element $\langle\psi_1^-|\mathcal{U}|\psi_2^-\rangle$

$$|\psi_2^-\rangle = \frac{1}{\sqrt{2}}(|110\rangle + i|001\rangle), \quad \langle\psi_2^-| = \frac{1}{\sqrt{2}}(\langle 110| - i\langle 001|).$$

$$U_{\text{rot}}|\psi_2\rangle = \frac{1}{\sqrt{16}}\sum_{i,j,k}\left[(-1)^{i+j} + i(-1)^k\right]|ijk\rangle.$$

$$\mathcal{U}|\psi_2^-\rangle = \frac{1}{\sqrt{16}}\sum_{i,j,k}\left[(-1)^{i+j} + i(-1)^k\right]e^{i\phi_{ijk}}|ijk\rangle,$$

$$\langle\psi_1^-|\mathcal{U}|\psi_2^-\rangle = \frac{1}{16}\left[(1+i)(-1-i)e^{i\beta} + (-i)(1+i)(1+i)e^{i\beta}\right]$$
$$= \frac{1}{16}\left[(-2i) + (2)\right]e^{i\beta} = \frac{1-i}{8}e^{i\frac{\kappa}{6}}.$$

Element $\langle\psi_1^-|\mathcal{U}|\psi_3^-\rangle$

$$|\psi_3^-\rangle = \frac{1}{2}(|100\rangle + |010\rangle + i|101\rangle + i|011\rangle), \quad \langle\psi_3^-| = \frac{1}{2}(\langle 100| + \langle 010| - i\langle 101| - i\langle 011|).$$

$$U_{\text{rot}}|\psi_3^-\rangle = \frac{1}{\sqrt{32}}\sum_{i,j,k}\left[(1+i)(-1)^i + (1=i)(-1)^j\right]|ijk\rangle.$$

$$\mathcal{U}|\psi_3^-\rangle = \frac{1}{\sqrt{32}}\sum_{i,j,k}\left[(1+i)(-1)^i + (1+i)(-1)^j\right]e^{i\phi_{ijk}}|ijk\rangle.$$

$$= \frac{1}{32}\left[(1+i)(1+i)e^{i\beta} + (1+i)(1+i)e^{i\gamma} + (-i)(1-i)(1-i)e^{i\delta}\right.$$
$$\left.+(-i)(1-i)(1-i)e^{i\beta}\right.$$
$$= \frac{i}{8}\left[2e^{i\frac{\kappa}{6}} + e^{i\frac{2\kappa_0-\kappa}{6}} - e^{-i\frac{\kappa}{6}}\right].$$

Element $\langle\psi_2^-|\mathcal{U}|\psi_1^-\rangle$

$$= \frac{1}{16}\left[(1+i)(1-i)e^{i\beta} + (-i)(1+i)(1-i)e^{i\beta}\right] = \frac{1-i}{4}e^{i\frac{\kappa}{6}}$$

.

Element$\langle\psi_2^-|\mathcal{U}|\psi_2^-\rangle$

$$= \frac{1}{16}\left[(1+i)(1+)e^{i\beta} + (-i)(1+)(1+i)e^{i\beta}\right] = \frac{1+i}{8}e^{i\frac{\kappa}{6}}$$

.

Element $\langle\psi_2^-|\mathcal{U}|\psi_3^-\rangle$

$$\begin{aligned}= \frac{1}{32}\left[(1+i)(1+i)e^{i\beta} + (1+i)(1+i)e^{i\gamma} + (-i)(1-i)(1-i)e^{i\delta}\right.\\ \left.+(-i)(1-i)(1-i)e^{i\beta}\right.\\ = \frac{i}{8}\left[2e^{i\frac{\kappa}{6}} + e^{i\frac{2\kappa_0-\kappa}{6}} - e^{-i\frac{\kappa}{6}}\right].\end{aligned}$$

Element $\langle\psi_3^-|\mathcal{U}|\psi_1^-\rangle$

$$\begin{aligned}= \frac{1}{16}\left[(1+i)(1-i)e^{i\beta} + (1+i)(1-i)e^{i\gamma} + (-i)(1-i)(1-i)e^{i\delta}\right.\\ \left.+(-i)(1-i)(1-i)e^{1\beta}\right.\\ = \frac{1}{8}\left[(1-i)e^{i\frac{\kappa}{6}} + (1-i)e^{i\frac{2\kappa_0-\kappa}{6}} + ie^{-i\frac{\kappa}{6}} + ie^{i\frac{\kappa}{6}}\right]\end{aligned}$$

.

Element $\langle\psi_3^-|\mathcal{U}|\psi_2^-\rangle$

$$\begin{aligned}= \frac{1}{16}\left[(1+i)(1+i)e^{i\beta} + (1+i)(1+i)e^{i\gamma}\right.\\ \left.+(-i)(1-i)(1-i)e^{i\delta} + (-i)(1-i)(1-i)e^{i\beta}\right.\\ = \frac{i}{8}\left[2e^{i\frac{\kappa}{6}} + e^{i\frac{2\kappa_0\kappa}{6}} - e^{-i\frac{\kappa}{6}}\right]\end{aligned}$$

.

Element $\langle\psi_3^-|\mathcal{U}|\psi_3^-\rangle$

$$= \frac{1}{32}\left[(1+i)^2e^{i\beta} + (1+i)^2e^{i\gamma} + (-i)(1-i)^2e^{i\delta} + (-i)(1-i)^2e^{i\beta}\right]$$

$$= \frac{i}{8}\left[2e^{i\frac{\kappa}{6}} + e^{i\frac{2\kappa_0-\kappa}{6}} - e^{-i\frac{\kappa}{6}}\right]$$

.

Final Matrix $\mathcal{U}_-$

Combining all elements, the matrix is:

$$\mathcal{U}_- = \frac{1}{4}\begin{pmatrix} \frac{1-i}{4}e^{-i\frac{2\kappa_0+\kappa}{6}} & \frac{1-i}{8}e^{i\frac{\kappa}{6}} & \frac{i}{8}\left(2e^{i\frac{\kappa}{6}} + e^{i\frac{2\kappa_0-\kappa}{6}} - e^{-i\frac{\kappa}{6}}\right) \\ \frac{1-i}{4}e^{i\frac{\kappa}{6}} & \frac{1+i}{8}e^{i\frac{\kappa}{6}} & \frac{i}{8}\left(2e^{i\frac{\kappa}{6}} + e^{i\frac{2\kappa_0-\kappa}{6}} - e^{-i\frac{\kappa}{6}}\right) \\ \frac{1}{8}\left((1-i)e^{i\frac{\kappa}{6}} + (1-i)e^{i\frac{2\kappa_0-\kappa}{6}} + ie^{-i\frac{\kappa}{6}} + ie^{i\frac{\kappa}{6}}\right) & \frac{i}{8}\left(2e^{i\frac{\kappa}{6}} + e^{i\frac{2\kappa_0-\kappa}{6}} - e^{-i\frac{\kappa}{6}}\right) & \frac{i}{8}\left(2e^{i\frac{\kappa}{6}} + e^{i\frac{2\kappa_0-\kappa}{6}} - e^{-i\frac{\kappa}{6}}\right) \end{pmatrix}.$$

This matrix shows the dynamics in the negative parity subspaces for the κ_0 and κ.

Chapter 8

Computation of Reduced Density Matrix

8.1 Initial State Selection

To study the evolution of entanglement, we consider an initial state that has components in both parity sectors of the system.

$$|\psi_0\rangle = \frac{1}{\sqrt{2}}\left(|\psi_1^+\rangle + |\psi_1^-\rangle\right) = \frac{1}{\sqrt{2}}\left(\frac{1}{\sqrt{2}}(|000\rangle - i|111\rangle) + \frac{1}{\sqrt{2}}(|000\rangle + i|111\rangle)\right) = |000\rangle.$$

So, we get the initial state as:

$$|\psi_0\rangle = |000\rangle.$$

8.2 Time Evolution Under the Floquet Operator

The system evolves under the Floquet operator $\mathcal{U}$, which is block-diagonal due to parity symmetry:

$$\mathcal{U} = \begin{pmatrix} \mathcal{U}_+ & 0 \\ 0 & \mathcal{U}_- \end{pmatrix}.$$

[2] state after n kicks is:

$$|\psi_n\rangle = \mathcal{U}^n|\psi_0\rangle.$$

As $|\psi_0\rangle$ lies in both parity subspaces, we have,

$$|\psi_0\rangle = \frac{1}{\sqrt{2}}\left(|\psi_1^+\rangle + |\psi_1^-\rangle\right),$$

that implies:

$$|\psi_n\rangle = \frac{1}{\sqrt{2}}\left(\mathcal{U}_+^n|\psi_1^+\rangle + \mathcal{U}_-^n|\psi_1^-\rangle\right).$$

8.3 Decomposition in the Parity Basis

We have,

$$\mathcal{U}_+^n|\psi_1^+\rangle = \sum_{i=1}^{3} c_i^+|\psi_i^+\rangle, \quad \mathcal{U}_-^n|\psi_1^-\rangle = \sum_{i=1}^{3} c_i^-|\psi_i^-\rangle,$$

therefore, the total state becomes:

$$|\psi_n\rangle = \frac{1}{\sqrt{2}} \sum_{i=1}^{3} \left(c_i^+|\psi_i^+\rangle + c_i^-|\psi_i^-\rangle\right).$$

Here,

- $\{|\psi_i^+\rangle\}$ form an orthonormal basis for he positive parity subspace.
- $\{|\psi_i^-\rangle\}$ form an orthonormal basis for the negative parity subspaces.
-

8.4 State in the Computational Basis

Using parity basis states in the Computational basis, the full state becomes:

$$|\psi_n\rangle = \sum_{x\in\{000,\ldots,111\}} a_x|x\rangle,$$

with amplitudes a_x expressed as:

$$\begin{aligned}
a_{000} &= \frac{1}{2}(c_1^+ + c_1^-), & a_{111} &= \frac{i}{2}(c_1^- - c_1^+),\\
a_{110} &= \frac{1}{2}(c_2^+ + c_2^-), & a_{001} &= \frac{i}{2}(c_2^- - c_2^+),\\
a_{100} &= \frac{1}{2\sqrt{2}}(c_3^+ + c_3^-), & a_{010} &= \frac{1}{2\sqrt{2}}(c_3^+ + c_3^-),\\
a_{101} &= \frac{i}{2\sqrt{2}}(c_3^- - c_3^+), & a_{011} &= \frac{i}{2\sqrt{2}}(c_3^- - c_3^+).
\end{aligned}$$

8.5 Reduced Density Matrix for Qubit 1

The density matrix is:

$$\rho_n = |\psi_n\rangle\langle\psi_n|,$$

and the reduced density matrix for the first qubit is obtained by tracing out qubits 2 and 3:

$$\rho_n^{(1)} = \text{Tr}_{2,3}(\rho_n).$$

The diagonal elements are:

$$\rho_{00}^{(1)} = \sum_{j,k=0}^{1} |a_{0jk}|^2 = |a_{000}|^2 + |a_{001}|^2 + |a_{010}|^2 + |a_{011}|^2,$$

$$\rho_{11}^{(1)} = \sum_{j,k=0}^{1} |a_{1jk}|^2 = |a_{100}|^2 + |a_{101}|^2 + |a_{110}|^2 + |a_{111}|^2.$$

The off-diagonal element is:

$$\rho_{01}^{(1)} = \sum_{j,k=0}^{1} a_{0jk} a_{1jk}^*, \quad \rho_{10}^{(1)} = (\rho_{01}^{(1)})^*.$$

Thus, the reduced density matrix is:

$$\rho_n^{(1)} = \begin{pmatrix} \rho_{00}^{(1)} & \rho_{01}^{(1)} \\ \rho_{10}^{(1)} & \rho_{11}^{(1)} \end{pmatrix}.$$

8.6 Entangle Quantification and Periodicity analysis via Linear Entropy via phase accumulation for Different Cases

The periodicity of the three-qubit kicked top system can be studied by examining the effective phase accumulated through the interaction Hamiltonian. The Floquet operator is defined as:

$$U = U_{\text{int}} U_{\text{rot}},$$

the interaction term is:

$$U_{\text{int}} = \exp\left[-i\frac{1}{6}(\kappa_0 \sigma_z^1 \sigma_z^2 + \kappa_0 \sigma_z^2 \sigma_z^3 + \kappa \sigma_z^3 \sigma_z^1)\right].$$

The product of the Pauli-$\sigma_z \sigma_z$ acts diagonally on the computational basis, having the eigenvalues ± 1. Therefore the effective phase acquired under $_{textint}$ can be written as,

$$\theta = \frac{1}{6}(2\kappa_0 + \kappa).$$

The phase θ studies the periodicity of the state evolution. When the accumulated total phase returns to 2π modulo 2π, that's when the state exhibit the periodicity

$$n\theta = 2\pi m \quad \Rightarrow \quad n = \frac{2\pi m}{\theta}, \quad m \in Z.$$

Therefore, the period T of the system is expressed as the smallest integer n that satisfies the above condition. This identifies the periodic points in the entanglement dynamics, verified by numerical simulations of the linear entropy $S_L(n)$.

The linear entropy is given by:

$$S_L = 1 - \mathrm{Tr}\left[(\rho_n^{(1)})^2\right]$$

We consider different values of κ_0, κ, and n as described below:

Case 1: $\kappa_0 = \pi$**,** $\kappa = 2\pi$**,** $n = 3$

$$\theta = \frac{1}{6}(2\kappa_0 + \kappa) = \frac{4\pi}{6} = \frac{2\pi}{3}$$
$$n\theta = 3 \cdot \frac{2\pi}{3} = 2\pi \Rightarrow \text{Periodic}$$

After 3 kicks, the system returns to the initial state:

$$|\psi_3\rangle = |000\rangle \Rightarrow \rho^{(1)} = |0\rangle\langle 0| = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \Rightarrow S_L = 0$$

[3, 9]

Case 2: $\kappa_0 = \frac{\pi}{2}$**,** $\kappa = \pi$**,** $n = 6$

$$\theta = \frac{1}{6}(2 \cdot \frac{\pi}{2} + \pi) = \frac{2\pi}{6} = \frac{\pi}{3}$$
$$n\theta = 6 \cdot \frac{\pi}{3} = 2\pi \Rightarrow \text{Periodic}$$

At $n = 6$, the system returns to the initial state:

$$|\psi_6\rangle = |000\rangle \Rightarrow \rho^{(1)} = |0\rangle\langle 0| \Rightarrow S_L = 0$$

Case 3: $\kappa_0 = \frac{2\pi}{5}$**,** $\kappa = \frac{2\pi}{5}$**,** $n = 15$

$$\theta = \frac{1}{6}(4\pi/5) = \frac{2\pi}{15}$$
$$n\theta = 15 \cdot \frac{2\pi}{15} = 2\pi \Rightarrow \text{Periodic}$$

At $n = 15$, we again return to the pure state:

$$|\psi_{15}\rangle = |000\rangle \Rightarrow \rho^{(1)} = |0\rangle\langle 0| \Rightarrow S_L = 0$$

Case 4: $\kappa_0 = \frac{\pi}{3}$**,** $\kappa = \frac{4\pi}{3}$**,** $n = 2$

$$\theta = \frac{1}{6}\left(2 \cdot \frac{\pi}{3} + \frac{4\pi}{3}\right) = \pi$$
$$n\theta = 2 \cdot \pi = 2\pi \Rightarrow \text{Periodic}$$

At $n = 2$, the system returns to its initial state:

$$|\psi_2\rangle = |000\rangle \Rightarrow \rho^{(1)} = |0\rangle\langle 0| \Rightarrow S_L = 0$$

Case 5: $\kappa_0 = \frac{2\pi}{7}$**,** $\kappa = \frac{6\pi}{7}$**,** $n \approx 8.4$

$$\theta = \frac{1}{6}\left(2 \cdot \frac{2\pi}{7} + \frac{6\pi}{7}\right) = \frac{10\pi}{42} = \frac{5\pi}{21}$$
$$n = \frac{2\pi}{\theta} = \frac{2\pi}{5\pi/21} = \frac{42}{5} = 8.4 \Rightarrow \text{Not exact integer}$$

The system does not return exactly to the initial state, so the reduced density matrix is mixed:

$$\rho^{(1)} \neq |0\rangle\langle 0| \Rightarrow S_L > 0 \quad \text{(e.g., } S_L \approx 0.12 \text{ at } n = 8)$$

Chapter 9

Results and Discussions

We analyze the entanglement dynamics of a three-qubit Floquet system under various values of κ_0 and κ. The entanglement is quantified using the **linear entropy** S_L of the reduced density matrix of qubit 1. The system is evolved for n kicks using the Floquet operator U, and we observe whether it returns to the initial state $|000\rangle$, indicating periodic behavior.

Case 1: $\kappa_0 = \pi$**,** $\kappa = 2\pi$**,** $n = 3$

After 3 kicks, the evolved state is:

$$|\psi_3\rangle = |000\rangle \Rightarrow a_{000} = 1, \quad a_x = 0 \text{ for all } x \neq 000$$

The reduced density matrix of qubit 1 is:

$$\rho^{(1)} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}, \quad S_L = 0$$

The system returns to the initial product state. Entanglement vanishes. This confirms exact periodicity.

Case 2: $\kappa_0 = \frac{\pi}{2}$**,** $\kappa = \pi$**,** $n = 6$

After 6 kicks, the state again returns to:

$$|\psi_6\rangle = |000\rangle \Rightarrow a_{000} = 1, \quad a_x = 0 \text{ for all } x \neq 000$$

$$\rho^{(1)} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}, \quad S_L = 0$$

This also shows perfect periodicity and zero entanglement at $n = 6$.

Case 3: $\kappa_0 = \frac{2\pi}{5}$, $\kappa = \frac{2\pi}{5}$, $n = 15$

Here too, the system returns to the initial state:

$$|\psi_{15}\rangle = |000\rangle \Rightarrow \rho^{(1)} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}, \quad S_L = 0$$

Although the period is longer, exact revival and disentanglement occur at $n = 15$. The dynamics remain regular and predictable.

Case 4: $\kappa_0 = \frac{\pi}{3}$, $\kappa = \frac{4\pi}{3}$, $n = 2$

This is a short-period system. The state evolves to:

$$|\psi_2\rangle = |000\rangle \Rightarrow \rho^{(1)} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}, \quad S_L = 0$$

The system oscillates with a 2-step periodic cycle and returns to a disentangled state at even values of n.

Case 5: $\kappa_0 = \frac{2\pi}{7}$, $\kappa = \frac{6\pi}{7}$, $n = 8$

In this case, the effective phase does not lead to a rational multiple of 2π. After 8 kicks:

$$|\psi_8\rangle \neq |000\rangle, \quad \rho^{(1)} \text{ is mixed}, \quad S_L \approx 0.12$$

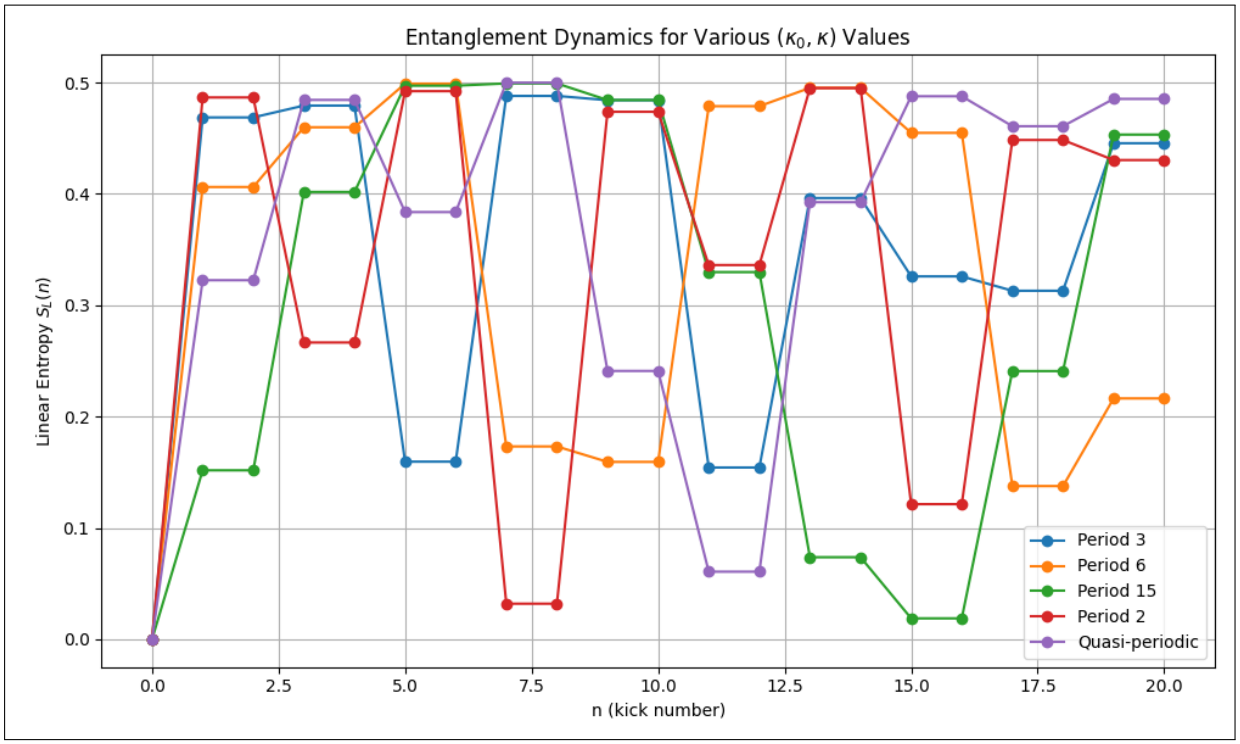
The system shows quasi-periodic behavior and does not return to a pure product state. Some entanglement persists, indicating complex or chaotic dynamics.

Summary Table

Case	κ_0	κ	**Periodicity**	$S_L(n)$
1	π	2π	Yes, $n = 3$	0
2	$\frac{\pi}{2}$	π	Yes, $n = 6$	0
3	$\frac{2\pi}{5}$	$\frac{2\pi}{5}$	Yes, $n = 15$	0
4	$\frac{\pi}{3}$	$\frac{4\pi}{3}$	Yes, $n = 2$	0
5	$\frac{2\pi}{7}$	$\frac{6\pi}{7}$	No (Quasi)	> 0

Thus, periodic cases exhibit a return to pure product states and zero entropy, while non-periodic (quasi-periodic) [7] cases yield nonzero entanglement.

9.1 Graphical Representations



- The figure shows the evolution of linear entropy$S_L(n)$ for qubit 1 in a three-qubit kicked top system under different parameter sets,κ_0,κ. Each curve corresponds to a distinct case exhibiting exact periodicity, and others quasi-periodic dynamics. For periodic cases(e.g, Period 2,3,6, and 15), the linear entropy returns to zero at regular intervals, indicating revival of the pure initial state $|000\rangle$ and the absence of entanglement. These periodic dips in S_L reflects the exact recurrence of the full system's wavefunction due to rational Floquet phases. In contrast, the quasi-periodic case (purple curve) never returns to zero, and the entropy fluctuations are irregular. This indicates that the system does not return to a product state and retains residual entanglement of irrational Floquet phase accumulation, leading to incommensurate evolution cycles.

CONCLUSIONS

This confirms that for he specific parameter values $\kappa_0 = \pi$, $\kappa = 2\pi$ and n=3, the system undergoes periodic evolution and returns to its initial pure, unentangled state. This behavior aligns with the analytical condition derived earlier, reinforcing its validity. By systematically varying the values of κ_0 and observing the time evolution of the state, we can investigate how entanglement emerges and how it periodically manifests in the dynamics of the system. Such an analysis provides deeper insight into the relationship between the system's parameters and its entanglement structure over time. Cases 1-4 confirm that the kicked top model exhibits periodic dynamics for rational phase accumulation, resulting in disentanglement at specific steps. Case 5 demonstrates quasi-periodic behavior and residual entanglement, marking a transition toward more complex or chaotic quantum dynamics. By plotting $S_L(n)$ for different κ_0, κ, and time steps n, we can study the evolution of the entanglement and periodicity that arise in the system.

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9.2 Python code for Graph

```
# Install scipy (only needed once)
!pip install scipy

# Import libraries
import numpy as np
import matplotlib.pyplot as plt
from scipy.linalg import expm

# Define Pauli matrices
I = np.eye(2)
sx = np.array([[0, 1], [1, 0]])
sy = np.array([[0, -1j], [1j, 0]])
sz = np.array([[1, 0], [0, -1]])

def kron3(a, b, c):
    return np.kron(np.kron(a, b), c)

def get_U(k0, k):
    # Rotation operator
    Urot = expm(-1j * np.pi / 4 * (kron3(sy, I, I) + kron3(I, sy, I)
        + kron3(I, I, sy)))

    # Interaction operator
    Uint = expm(-1j / 6 * (k0 * kron3(sz, sz, I) + k0 * kron3(I, sz,
        sz) + k * kron3(sz, I, sz)))

    return Uint @ Urot

def reduced_density_matrix(psi):
    rho = np.outer(psi, np.conj(psi))
    rho1 = np.zeros((2, 2), dtype=complex)
    for i in range(2):
        for j in range(2):
            for k in range(4):
                rho1[i, j] += rho[i*4 + k, j*4 + k]
    return rho1

def linear_entropy(rho1):
    return 1 - np.trace(rho1 @ rho1).real

def simulate_entropy(k0, k, n_max):
    U = get_U(k0, k)
    psi = np.zeros(8, dtype=complex)
    psi[0] = 1 # |000>
    entropies = []
    for n in range(n_max + 1):
        psin = np.linalg.matrix_power(U, n) @ psi
        rho1 = reduced_density_matrix(psin)
```

```
        S = linear_entropy(rho1)
        entropies.append(S)
    return entropies

# Define (k0, k, label)
cases = [
    (np.pi, 2*np.pi, "Period 3"),
    (np.pi/2, np.pi, "Period 6"),
    (2*np.pi/5, 2*np.pi/5, "Period 15"),
    (np.pi/3, 4*np.pi/3, "Period 2"),
    (2*np.pi/7, 6*np.pi/7, "Quasi-periodic")
]

n_max = 20
plt.figure(figsize=(10, 6))

for k0, k, label in cases:
    entropies = simulate_entropy(k0, k, n_max)
    plt.plot(range(n_max + 1), entropies, label=label, marker='o')

plt.xlabel("n (kick number)")
plt.ylabel("Linear Entropy $S_L(n)$")
plt.title("Entanglement Dynamics for Various $(\\kappa_0, \\kappa)$
    Values")
plt.legend()
plt.grid(True)
plt.tight_layout()
plt.show()
```